GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

MATHEMATICAL MODELING OF CONCENTRATION OF CONTAMINANT LEVEL FORWATER INDAMODAR RIVER BERMO REGION

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ABSTRACT

To evaluate the environment impact of water pollution ofDamodar river mathematical models play a major role in predicting the pollution level in regions under considering.This paper we discuss the various mathematical models involving water pollutant transport equation inDamodarriver in Jharkhand. We also give the implicit central difference scheme in space and a forward difference method in time for evaluation of the generalized transport equation. In this paper we also developed different mathematical model of concentration of contaminant level for water in DamodarRiver at different places in Jharkhand.

Keywords: *Water pollution, Damodarriver, dissolved Oxygen, Biological oxygen demand, Contaminant transport level, water transport.*

I. INTRODUCTION

There are several companies which perform coal mining activities and is located on the banks of DamodarRiver. Damodar river basin is repository of 40% of India coal reserve. Exploitation of and related industries in the area has exerted a great impact on the water pollution [8] Mibagheriet. Al developed mathematical model of water quality in river system in JajroadRiver[9] Shuklaet. Al developed mathematical model and studied its analysis of the depletion of dissolved oxygen in entropies water bodies affected by organic pollutants. Pollution is also caused when silt and other suspended solids such as soil, washoffplowed field, Construction and logging sites, urban areas, eroded river banks when natural bacteria and protozoan in the water break down this organic material. They begin to use up the oxygen dissolved in the water. Let us are another type of pollution that prove very harmful. They can cause many illnesses that range from typhoid and dysentery to major respiratory and skin diseases. Pathogens included such organism as bacteria, viruses and protozoa. These pollutants enter waterway through untreated sewage, storm drains, septic tanks and particularly boats that dump sewage. Through microscopic these pollutants have a tremendous effect evidenced by their ability to cause sickness. Oxygen is required to support aquatic life and maintain water equality. It is the most important dissolved gas in water. Water is equilibrium with air at 25 degree contain 8.3 mg/ L of dissolved O_2 . Although water molecules contain an oxygen atom, this oxygen in not what is need by aquatic organisms living in natural waters. A small amount of oxygen up to ten molecules of oxygen per million of water is actually dissolved in water. Fish and Zooplankton breath dissolved oxygen and without sufficient oxygen mortality will occur Dissolved oxygen (DO) concentrations are effectedby a number of factors. Higher Do is produced by turbulent actions such as waves, such mix air and water. Lower water temperatures also allows for retention of higher do concentrations. Low Do levels tend to occur more often in warmer, slow moving waters. In general low Do concentrations may be high near the surface due to wind action and plant photosynthesis, but may be entirely depleted (amoxic) at the bottom. Oxygen consuming wastes include decomposing organic matter or chemicals that reduce DO in the water. Raw domestic waste water contains high concentration of oxygen subsuming wastes that need to be removed before it can be discharge into a waterway. Maintaining a sufficient level of Do in water is Critical to most forms of aquatic life. Microorganisms such as bacteria are responsible for decomposing organic water when organic matter such as dead plants, leaves, grass clippings, manure, sewage or even food waste is present in a water supply the bacteria will begin the process of breaking down this waste, when this happens much of the available dissolved oxygen is consumed by aerobic bacteria, robbing other aquatic organisms of the oxygen they need to live.

Biological oxygen Demand (BOD) is an indicator for the concentration of bio gradable organic used to infer the general quality for the water and its degree of pollution. BOD measures the rate of uptake of oxygen by micro organisms and over a given period of time. Dissolved organic water (DOM) is widely present in aquatic subsurface environments. Its contains many biochemically identifiable classes of compounds such as sugar or amina acids as well as fractiousthat are more coarsely classified such as humus. To evaluate the environmental impact or pollution. Mathematical models play a major role in predicting the pollution level in the regions under consideration [1, 2,3,12, 13, 17]

In the surface colloidal / bacterial particles are generated and / or mobilized by various mechanisms. The presence of colloids cans the transport behavior of organic contaminants in soils and groundwater due to sorption on the surface of colloids / bacteria. [3, 5, 6]

In riverbank filtration, the mobile colloidal particles can crease the mobility of contaminants and the degree of sorption and microbial degradation several researches have reported that organic compounds have strong affinity to DOM. Massett and Andersiom have reported that hydrophobic organic compound such as PCBS were bound to DOM in water. Center and suffethave examined the sorption and DDT to DOM. Chiou, et al have shown that the sorptionof hydrophobic organic compounds onto DOM increased their aqueous solubility. Magee et al. have reported that hydrophobic compounds moved faster in the presence of DOM. In riverbank filtration, DOM can facilitate the contaminant transport as the mobile carrier. In addition, it can be utilized as a food source for bacteria.In riverbank filtration; contaminant transport can be affected by the presence of bacteria. Lindqvist and Enluiedhave reported that the transport of DDT and hexachorobenzene could be facilitated in groundwater owing to their attachment on bacteria. Jenkins and lion have demonstrated that highly mobile bacteria could increase the mobility of organic.Contaminants such as PAMS. In addition bacteria can reduce the contaminants concentration by microbial transformations. Kim and carapcioglugave a model to simulate contaminant transport in riverbank filtration in the presence of DOM and bacteria. The model equations are solved numerical with a fully implicit finite difference method.

Nomenclature

θ : Water content
n : Porosity
σ_b : Volumetricfraction of Bacteria
C_b : Concentration of Aqueous phase Bactria
C_d : Concentration DOM in the Aqueous phase
D_b : Hydrodynamic dispersion coefficient forbacteria
σ_{cd} : Mass fraction of the contaminant sorbet to DOM
σ_{cbm} : Mass fraction of the contaminant sorbet to the mobile bacteria
ρ_b : Density of bacteria
k_c& k_b : Deposition and release rate coefficient of bacteria on the solid matrix
S_t : Storage coefficient of the aquifer dimensionless
h : Hydraulic head
W(x,t) : Volume flux per unit area
S : ContaminantConcentration Sorbed to soil.
C : Contaminant cone
B : Bulk Density
W : Porosity
D : Hydrodynamic dispersion tensor
V : fluid velocity vector
K_w : Composite equilibrium sorption coefficient
K : Overall mass transfer coefficient
β_i : Diffusivity term
R_i : Biochemical reaction

S_i : External source
K_i : Kinetic constant
ρ_1 : Concentration of Biochemical demand
ρ_2 : Concentration ofDO
$\delta(P_i)$: Dicra measure

II. WATER TRANSPORT

If only Isothermal contaminant transport in the saturated zone of the surface is consider. The general physical law equation for the saturated water transport through a representative small volumetric element in the porous structure of the surface is given by

$$S_t \frac{\partial h}{\partial t} + W(x,t) = \nabla.(K_{sat.} \nabla h) \tag{1}$$

III. CONTAMINANT TRANSPORT

The general physical chemical law Convective-dispersive-reaction contaminant transport equation for this same element of volume for a single chemical is given by

$$\frac{\partial}{\partial t}\left(C + \frac{\rho\beta}{\in}\right) = \nabla.(D.\nabla C)(x,t) - V.\nabla C - \Lambda^{\omega} C^{\omega} - \frac{S_s}{\in}\frac{Q}{vol} \tag{2}$$

For equilibrium sorption approximated by a linear.Freundlich isothermal the amount absorbed to solid component of the subsurface is approximated by

$$S = K_w C \tag{3}$$

IV. CONTAMINANT TRANSPORT MODELS:

Describing Contaminant transport requires the solution of the equation for saturated water transport and the convection - dispersion-reaction equation for the Contaminant transport through porous Media. Microscopic Contaminant transport biological & chemical transformation is first order reaction

$$R_f \frac{\partial_{sc}}{\in} = \nabla.(D.\nabla C) - V.\nabla C - \Lambda - \frac{S_s}{\in}\frac{Q}{vol} \tag{4}$$

$$R_f = 1 + \frac{\rho\beta K_{\omega}}{\in} \tag{5}$$

If Sorption is not an equalitarian process it can be described by an interphase mass transfer process as

$$S = K_w C \qquad (6)$$

The retardation factor

$$R_f = 1 + \left[\frac{\rho \beta K_\omega}{\in} K C^{\omega - 1} \right] \qquad (7)$$

General form of the cone profile for Contaminant transport in the Subsurfaceiis given as function of these transport parameter and subsurface characteristic as

$$C(x, y, z, t) = F\left(D, K_{sat.} \nabla, k, \in, \rho\beta\right) \qquad (8)$$

V. BACTERIA TRANSPORT

Waste contains a great variety of pathogenic bacteria and viruses. We consider a mixture of N reacting species with partial densities $d_i \ , i = 1, 2 ....N$

Let d be the density of the mixture and Y_i the mass fracture of species *i*,

$$D = \sum_{i=1}^{N} d_i \, , \gamma = \frac{d_i}{d}, i = 1, 2....N$$

If V_i denotes the velocity of species i, the macroscopic velocity is defined by

$$V = \sum_{i=1}^{N} \gamma_i V_i$$

The difference between Vand Vican be decomposed in the migration velocity and diffusion velocity, the former representing $V - V_i = \sigma_i + V_i$

We can write from Onsager's law

$$d_i V_i = -\beta_i \nabla d_i \ , i = 1, 2........N$$

The mass conservation equation is

$$\frac{\partial d_i}{\partial t} + \nabla\left(d_i V\right) + \nabla\left(d_i \sigma_i\right) - \beta \Delta d_i = R_i + S_i \qquad (9)$$

After discharge, cone of bacteria's viruses decrease very quickly due to unfavorable conditions like lack of nutrients, low temperature, Sun rays, etc. Death rate is frequently modeled as a first order reaction this means that the R_i term in equation (9) given by

$$R_i = -K_i C \qquad (10)$$

[Chattopadhyay 5(8): August 2018] ISSN 2348 – 8034
DOI- 10.5281/zenodo.1327437 Impact Factor- 5.070

Tq_o, i i related to the relation $Tq_o, i\frac{\log 10}{K_i}$ and $S_i = \sum q_j\, r_i^j\, \delta\left(\rho_j\right)$

Equation (9) becomes

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2} - K_i C + \sum q_j\, r_i^j\, \delta\left(\rho_j\right) \quad (11)$$

VI. BIOLOGICAL OXYGEN DEMAND BOD/DISSOLVED OXYGEN (D.O):

Oxygen plays a major role in all kinds of life. In particular, it used by bacteria to decompose the organic matter. If the oxygen demand is not satisfied, disappear. However, decomposition of oxygenic matter goes on by anaerobic processes which do not use oxygen but produce sulfur of hydrogen and methane both having a mollusous smell. The organic matter can be measured in terms of the need of oxygen to decompose it, the so-called biological oxygen demand (BOD). If the pollution level is not too high this need can be satisfied by dissolved oxygen (DO), it can be express byclassical model given by Streeter and Phelps for the evolution of BOD and DO

$$\frac{\partial \rho_1}{\partial t} + u\nabla\rho_1 - \beta_1\Delta\rho_1 = -K_1\rho_1 + \frac{1}{h}\sum_{j=1}^{N} q_j\, r_i^j\, \delta\left(\rho_j\right) \quad (12)$$

And also

$$\frac{\partial \rho_2}{\partial t} + u\nabla\rho_2 - \beta_2\Delta\rho_2 = -K_1\rho_1 + \frac{1}{h}K_2\left(d_s - \rho_2\right) - \frac{1}{h}\gamma_p M + \frac{1}{h}\frac{I_\beta}{a + bI_\beta + cI_\beta^2}M + \frac{1}{h}F \quad (13)$$

If the quantity of organic matter increases beyond a maximum value the dissolved. Oxygen is not enough to decompose it leading to modification in the ecosystem.

VII. DISSOLVED ORGANIC MATTER (DOM):

Bacteria transport in presence of DOM: The movement of bacteria in soils and groundwater in manly controlled by the advective - dispersive. The mass balance equation for the aqueous -phase bacteria can described as

$$\frac{\partial\left(\theta C_b\right)}{\partial t} = -\frac{\partial}{\partial x}\left[-D_b\frac{\partial\left(\theta C_b\right)}{\partial x} + V_w\theta C_b\right] - K_c\theta C_b + K_\gamma\rho_b\sigma_b$$
$$+\mu\left(C_c + \frac{\rho_s K_1 C_c}{\theta} + C_d\sigma_{cd} + C_d\sigma_{cbm}\right)\theta C_b - K_{dm}\theta C_b - K_\theta\gamma\theta C_{cd} \quad (14)$$

VIII. CONTAMINANT TRANSPORT IN PRESENCE OF DOM

The mass balance equation of the contaminant dissolved in the aqueous phase can be expressed by

$$\frac{\partial(\theta C_c)}{\partial t}+\frac{\partial \rho_s \sigma_{cs}}{\partial t}=-\frac{\partial}{\partial x}\left[-D_c\frac{\partial(\theta C_b)}{\partial x}+V_w\theta C_b\right]$$

(15)

Where C_c is

$$-K_\rho\theta C_c-K_q\theta C_d\sigma_{cd}+K_3\theta C_c-K_4\theta C_b\sigma_{cbm}-K_c\theta C_b\sigma_{cbm}-\frac{\mu C_c}{\gamma}\left(\theta C_b+\rho_b\sigma_b\right)-\frac{\mu\rho_s\sigma_{cs}}{\gamma\theta}\left(\theta C_b+\rho_b\sigma_b\right)$$

If the sorption relationship between the aqueous phase and the solidmatrix in assumed to be an equilibrium-controlled process and represented by linear isotherm, the mass fraction of the contaminant sorbet into the solid matrix can be presented as

$$\sigma_{cs}=K_1 C_c$$

(16)

$$\frac{\partial(R_c\theta C_c)}{\partial t}=-\frac{\partial}{\partial x}\left[-D_c\frac{\partial(\theta C_c)}{\partial x}+V_w\theta C_c\right]$$

Where R_c is

$$-k_\rho-k_q\theta C_d\sigma_{cd}-2k_3\theta C_c-k_4\theta C_b\sigma_{cbm}-k_c\theta C_b\sigma_{cbm}-\frac{\mu R_c}{\gamma}\left(C_b+\frac{\rho_b\sigma_b}{\theta}\right)\theta C_c$$

we can represent contaminant retardation factor as follows

$$R_c=\frac{1+\rho_b k_1}{\theta}$$

(17)

The one dimensional transport equation for the contaminant sorbed to the DOM is

$$\frac{\partial(\theta C_d\sigma_{cd})}{\partial t}=-\frac{\partial}{\partial x}\left[-D_b\frac{\partial(\theta C_d\sigma_{cd})}{\partial x}+V_w\theta C_d\sigma_{cd}\right]-k_\rho\theta C_c-k_q\theta C_d\sigma_{cd}-\frac{\mu C_b}{\gamma}\left(\theta C_d\sigma_{cd}\right)$$

(18)

The one dimensional transport equation for the contaminant sorbed to the mobile bacteria

$$\frac{\partial(\theta C_b\sigma_{cbm})}{\partial t}=-\frac{\partial}{\partial x}\left[-D_b\frac{\partial(\theta C_b\sigma_{cbm})}{\partial x}+V_w\theta C_b\sigma_{cbm}\right]+k_3\theta C_c-k_4\theta C_b\sigma_{cbm}-\frac{\mu C_b}{\gamma}\left(\theta C_b\sigma_{cbm}\right) \quad (19)$$

IX. DIMENSIONAL TRANSPORT EQUATION

The one Dimensiontransport equation can be generalized as

$$\frac{\partial}{\partial t}C(x,t)+q\frac{\partial}{\partial x}C(x,t)+D\frac{\partial^2}{\partial x^2}C(x,t)+kC(x,t)=0 \qquad \text{in } Q$$

(20)

$$C(x,0) \qquad in\ \Omega$$

$$\frac{\partial}{\partial x}C(x,t) \qquad in\sum$$

The mass transport equation (17) can be solved by using the Crank-Nicolson finite difference scheme. The finite difference representation of equation (20) is for any point *i*,at any time *t* is

$$\frac{C_i^{j+1}-C_i^j}{\Delta t}+\frac{q}{2}\left[\frac{C_{i+1}^{j+1}-C_{i-1}^{j+1}}{2\Delta x}+\frac{C_i^{j+1}-C_i^j}{2\Delta x}\right]=-\frac{D}{2}\left[\frac{C_{i+1}^{j+1}-2C_i^{j+1}+C_{i-1}^{j+1}}{2\Delta x}+\frac{C_{i+1}^j-2C_i^j+C_{i-1}^j}{2\Delta x}\right]+kC_i^{j+1}=0$$

(21)

Thus we have

$$-\gamma C_{i-1}^{j+1}+(2-\delta+2\gamma+\gamma k)C_i^{j+1}+(\delta-\gamma)C_{i+1}^{j+1}=\gamma C_{i-1}^{j+1}+(2+\delta-2\gamma)C_i^j+(\gamma-\delta)C_{i+1}^j$$

(22)

Where $\delta=\frac{q\Delta t}{2\Delta x}$, $\gamma=\frac{D\Delta t}{\Delta x^2}$

boundary and initial condition are

$$C(x,o)=C_0 \text{ and } \frac{\partial C}{\partial x}=g(t)$$

In order to evaluate (22) at $i = 1$ and $i = n$ the boundary values C_0^j and C_{n+1}^j are needed.

We developed the difference equation

$$(1-u_1)C_0^j-u_1\left(\frac{C_1^j-C_0^j}{\Delta x}\right)=g_1(t_j)$$

(23)

$$(1-u_2)C_{n+1}^j-u_2\left(\frac{C_{n+1}^j-C_n^j}{\Delta x}\right)=g_2(t_j)$$

(24)

solving these equations , we et

$$C_0^j = \frac{g_1(t_j)\Delta x + u_1 C_1^j}{u_1 + (1-u_1)\Delta x}$$

(25)

$$C_{n+1}^j = \frac{g_2(t_j)\Delta x + u_2 C_n^j}{u_2 + (1-u_2)\Delta x}$$

(26)

The crank-Nicolson equation can be expressed in vector form be letting

$C^j = \left[C_1^j, C_2^j, C_3^j \ldots\ldots\ldots\ldots C_n^j\right]$ denote the solution at time t_k for $0 \le j \le k$. Suppose both boundary conditions are Neumann which means

$$u_1 = 1 \,,\, u_2 = 1$$

Suppose $A = (2-\delta+2\gamma+2K), B = (\delta-\gamma), C = (2+\delta-2\gamma), D = (\gamma-\delta)$

The stability of the method is controlled by the dispersion and advection current number, defined as

$$C_{adv} = \frac{V\Delta t}{\Delta x} \quad and \quad C_{disp} = \frac{D\Delta t}{\Delta x^2}$$

X. RESULTS AND DISCUSSION

We solve the contaminant transport model by implementing the crank-Nicolson numerical scheme, while varying different parameter values. We observe in general, a decrease in contaminant concentration. If fig.1, the profile for varying the decay rate, we observe that with a higher decay rate, the concentration of contaminant decreases faster than with a lower decay rate. In fig. 2 , the profile for varying contaminant velocity, we observe that the contaminant concentration with a higher velocity decreases at a higher rate than that with a lower velocity. In fig. 3.the profile for varying the diffusive term, we saw that when the rate of diffusion is high, the concentration of contaminant decreases faster.

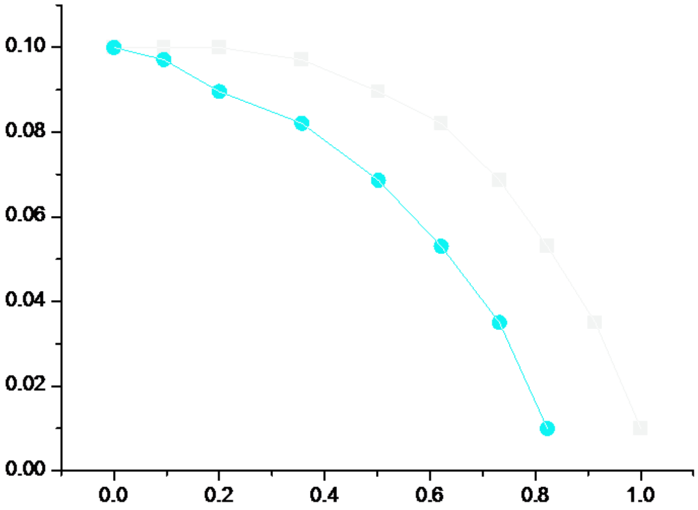

Figure. 1
Varying the value for the rate of decay "k"

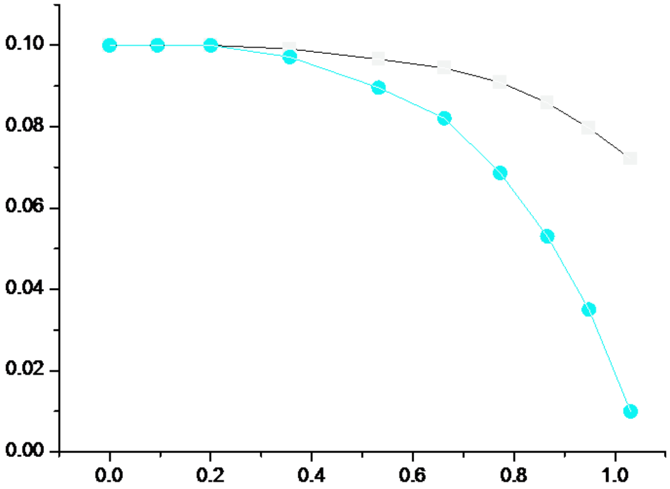

Figure. 2
Varying the value for the velocity term "q"

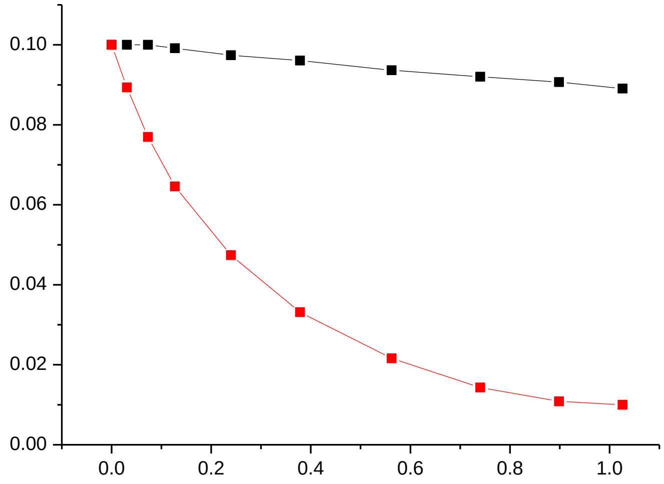

Figure. 3
Varying the value for the diffusive term "D"

XI. CONCLUSION

In this paper we have discussed the various Mathematical models involving water pollutant transport equation in Damodarriver. We have also given the implicit Central difference scheme in space and a forward difference method in time for the evaluation of the generalized transport equation and have given profiles for different parameter values.

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